Effective mass and other properties of remotely doped Ge quantum wells

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The aim of this report is to investigate the effective mass, the carrier density and other properties in remotely doped Ge layers relaxed on $Si_{0.3}Ge_{0.7}$ buffers. To determine the effective mass we have used the temperature dependency of the Shubnikov-de Haas (SdH) oscillations. The results showed that the effective mass to depends slightly on the applied (perpendicular) magnetic field. Two different methods were used to obtain the carrier density: The first approach uses the maxima and minima of the SdH oscillations, the second the linearity of the Hall resistance to the perpendicular magnetic field. Finally we investigate the dependency of the carrier density on the applied magnetic field and some other effects of the measurement.

I. INTRODUCTION

It is a well known fact and has been shown in different publications[1,2] that SiGe material systems greatly extend the performance of Si-based semiconductors. The aim of this report is to investigate the effective mass and the carrier density of device 7396-16, a remotely doped Ge quantum well. For more information about the structure of the device 7396-16 and its fabrication the reader is referred to the appropriate references[1]. In Sec. II the device and our first measurements using the instruments are described shortly. The determination of the effective mass of device 7396-16 can be found in Sec. III, where the fact is used that the envelopes of the Shubnikov-de Haas (SdH) oscillations at different temperatures give a useful tool to determine the effective mass. In Sec. IV we will measure the carrier density using two different approaches. While one of these approaches uses the distance between maxima or minima of the SdH oscillations to determine the carrier density, the other uses the relation between the hall resistance and the applied magnetic field. Additionally we show that the carrier density depends on the applied magnetic field for constant temperature. In Sec. V we make more comments about measurements and observations we've made e.g. the difference in the resistance for different orientations of the sample at vanishing perpendicular magnetic field.

II. MEASUREMENTS

A Hall bar as in Fig. 1 has been fabricated to measure the voltage $V_{||}$. Due to the formula $V_{||} = R_{||} \cdot I$, where I is the current applied over the hall bar, we obtain the resistance $R_{||}$. Additionally we measured $V_{\perp}$ to see the Hall effect and be able to determine the carrier density in an alternative way, see Sec. IV. The instrument used to make these measurements was a *Physical Properties Measurement System* (PPMS) and the accompanied software PPMS MultiVu. Through all the measurements we used a current of about $100nA$ and two Lock-In amplifiers. The obtained data has been written to the computer through a self made application. The numerical toolbox $Matlab$[3] has been chosen to take the task of data evaluation and graphical output. The programs written to obtain the results can be found in the Appendix A. To check whether we connected the contacts of the sample correctly with the sample holder, we measured the quantum hall effect and as the data we obtained looked good, we went into finding the effective mass.

III. THE EFFECTIVE MASS

A. Theory

A straightforward way to obtain the effective mass is the one proposed by Smrčka and Isihara[4]. The resistance $R_{||}$ can be expressed as

$$R_{||} = R_0(1 + 2\frac{\Delta g(T)}{g_0}) \tag{1}$$

where $g(T)$ is the density of states at temperature T, g_0 is the density of states at magnetic field $B = 0$ and R_0 the resistance at zero field. They showed that the fraction $\frac{\Delta g(T)}{g_0}$ can be expressed the following way

$$\frac{\Delta g(T)}{g_0} = 2\sum_{s=1}^{\infty} e^{\frac{-\pi s}{\omega_c \tau_q}} \frac{2\pi^2 s k_B T/\hbar\omega_c}{\sinh(2\pi^2 s k_B T/\hbar\omega_c)} \cos(\frac{2\pi s \epsilon_F}{\hbar\omega_c} - \pi s) \tag{2}$$

where k_B is the Boltzmann constant, T the temperature and $\omega_c = eB_{\perp}/m^*$ the cyclotron frequency of the holes. As we want to measure the effective mass m^*, there has to be considered only the second term in Eq. 2 which is the envelope of the SdH oscillation. The cosine and the exponential term are independent of the temperature what makes it needless to take them into account, as we will see. We've measured the resistance $R_{||}$ (rather the voltage $V_{||}$) at different magnetic fields $B_{\perp}$ (as we are only interested in the perpendicular term of the applied magnetic field B, see later) and fitted the temperature dependent amplitudes of the SdH oscillations at fixed $B_{\perp}$ to the formula $C\frac{x}{\sinh(x)}$ where $x = 2\pi^2 k_B T/\hbar\omega_c$. The fit

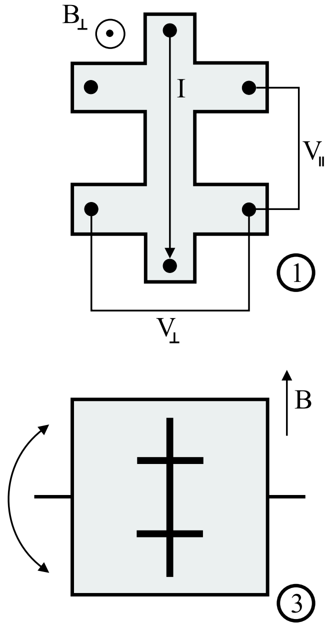

FIG. 1: 1) The fabricated hall bar that has been used to take the measurements. While the voltage $V_{||}$ leads to the SdH oscillations the voltage $V_\perp$ measures the quantum hall effect. 2) Visualization of the parallel and perpendicular field with respect to the sample holder in a rotated position. 3) First setup where the parallel magnetic field is in direction of the current I. This setup is called direction one in the report. 4) Second setup where the parallel magnetic field is perpendicular to the current I. This setup is called direction two in the report.

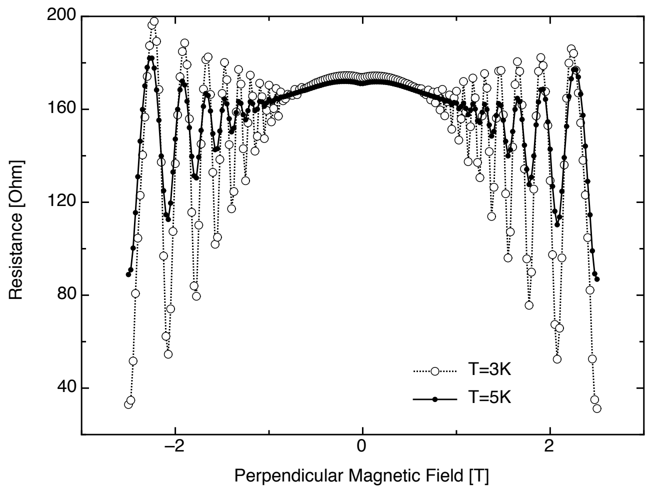

FIG. 2: Shubnikov-de Haas oscillations for direction one and temperature 3K and 5K.

parameters were m^* and C. The data the fit is applied to consists of all the maxima and minima[5] of one side of $B_\perp = 0$, means either $B_\perp < 0$ or $B_\perp > 0$. For a more detailed discussion of the fitting procedure, the reader is referred to the Appendix A.

B. Measurement

We have measured the resistance with respect to the applied field for two different orientations of the hall bar and for different perpendicular and parallel field $B_\perp$ and $B_{||}$ (see Fig. 1). First the sample has been set perpendicular to the applied magnetic field (this means $B_{||} = 0$) into the PPMS. We made measurements for different temperature $T = 2K \dots 6K$ and scanned the magnetic field in the range $B_\perp = -2.5T \dots 2.5T$. The obtained plot of the resistance with respect to the perpendicular magnetic field for different temperatures can be be seen in Fig. 2. The next task was to take the effect of a non-vanishing parallel magnetic field $B_{||}$ into account. The horizontal rotator of the PPMS set the sample in a $\pi/2$ angle with respect to the applied magnetic field into the PPMS. From this position we were able to rotate the sample in a range $[-\varphi, \varphi]$, where $B_\perp(-\varphi) = -2.5T$ and $B_\perp(\varphi) = 2.5T$ as can be seen in Fig. 3. The angle φ depends on the applied field B and the parallel magnetic field $B_{||}$ differs from the applied field B for φ not equal to zero. The difference of $B_{||}$ to B can be seen in Tab. I and the angles φ in dependency of the field B in Tab. II. One problem arouse during the analysis of the data: The larger the applied field was, the less SdH oscillations occurred what finally ended in bad fits for the effective mass. Due to this fact, there was only a limited range of $B_\perp$ where we could compare the different effective masses. At small fields $B_\perp$ there are no oscillations, so it's not possible to compute the effective mass for small perpendicular fields. We also couldn't measure the effective mass around $B_\perp = \pm 2.5$ with sensible error because of the lack of data points. This can be anticipated by scanning the perpendicular field not only to $B_\perp = \pm 2.5T$ but to higher/lower fields. The area of interest is therefor $B_\perp = 1.5T \dots 2.3T$. In Fig. 4 one can see the ratio of the effective mass and the electron mass in dependency of the perpendicular magnetic field $B_\perp$ and applied field B for orientation one. As Fig. 4 shows, the curves of the investigated ratio depends on the applied field i.e. the parallel field $B_{||}$. The higher $B_{||}$, the bigger the offset from the ratio at zero field. In Fig. 5 one can see the same but for orientation two and the applied magnetic field goes from $5T$ to $14T$ (as $14T$ is the maximum field the super-conducting magnet in the PPMS is able to produce). The vertical lines indicate the range, where there should be (almost) enough data points to compare the ratios. The vertical black lines in Fig. 4 and Fig. 5 at two different fields $B_\perp$ are the 95% confidence intervals of the fit for the effective mass. The measurements lead to the conclusion that the effective mass depends on the applied parallel field $B_{||}$. The conclusion that the effective mass depends on the perpendicular magnetic field $B_\perp$ can not be verified, because the confidence intervals are to large i.e. the effective mass could be a constant with respect to the perpendicular field $B_\perp$.

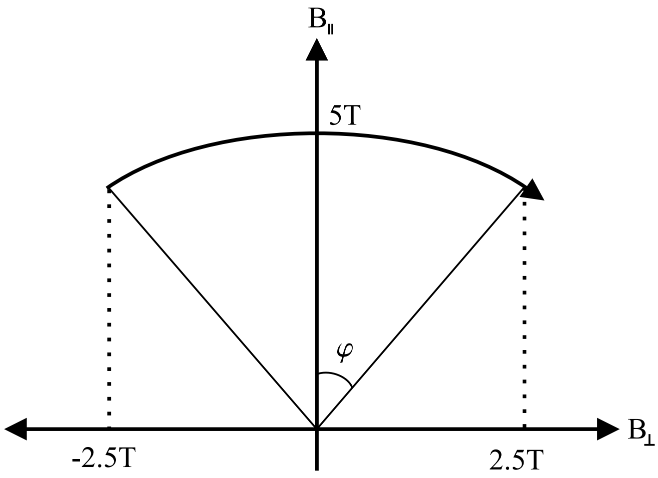

FIG. 3: Visualization of the scan procedure for non-vanishing parallel magnetic field $B_{||}$. While the perpendicular field $B_\perp$ is in a range from -2.5 to 2.5, the parallel field $B_{||}$ is not constant during the measurement (as it would be in a perfect measurement).

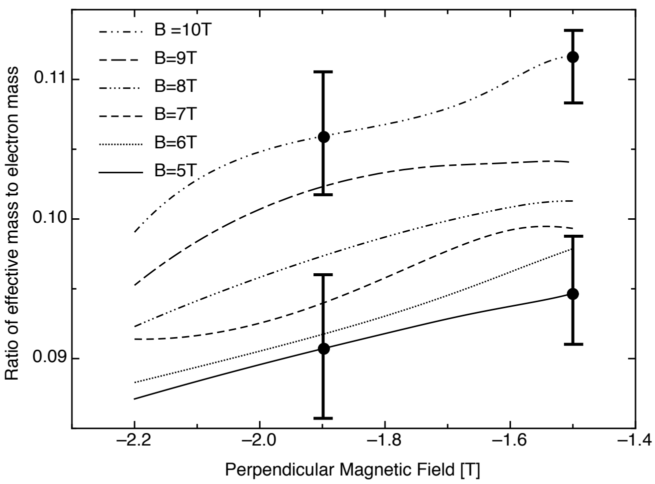

FIG. 4: The ratio of the effective mass to the electron mass in dependency of the applied field for direction one. The higher the parallel field the higher the average ratio gets. The confidence intervals are at $B_\perp = -1.5T$ and $B_\perp = -1.9T$ for $B = 5T$ and $B = 10T$

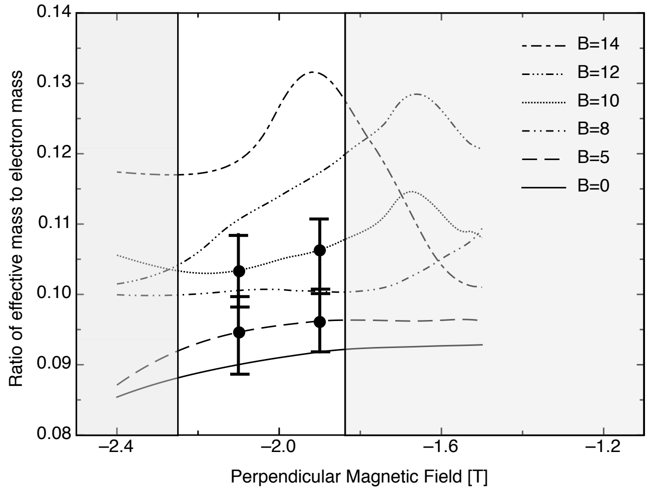

FIG. 5: The ratio of the effective mass to the electron mass in dependency of the applied field for direction two. The higher the parallel field the higher the average ratio gets. The grey fields indicate the range where there are not enough data points to compute the effective mass with sensible error (There aren't enough oscillations to obtain a good envelope). The confidence intervals are at $B_\perp = -1.9T$ and $B_\perp = -2.1T$ for $B = 5T$ and $B = 10T$

TABLE I: In this table one can see the maximum error of the parallel field as we turn it around the angle φ. $B_{||}^{\rm min}$ is the minimal value during the measurement and $B_{\rm rel} = (B_{||} - B_{||}^{\rm min})/B_{||}$ is the maximal relative error.

$B_{\|\|}$ [T]	$B_{\|\|}^{\rm min}$ [T]	$B_{\rm rel}$ [%]
5	4.33	13.4
6	5.45	9.1
7	6.54	6.6
8	7.60	5.0
9	8.65	3.9
10	9.68	3.2
11	10.71	2.6
12	11.74	2.2
13	12.76	1.9
14	13.77	1.6

IV. THE CARRIER DENSITY p

A very important quantity in modern semiconductors is the carrier density p. From our data, there are two possible methods of measuring the carrier density in the device using the PPMS.

A. Method using Shubnikov-de Haas Oscillations

As we have used the SdH oscillations before to determine the effective mass using the envelope of the amplitudes, we use them now to determine the carrier density p. It is a fact that we can write the Landau levels $\nu = 0, 1, 2, \ldots$ of a sample in a magnetic field B as

$$\nu = \frac{ph}{2e} \cdot \frac{1}{B_\perp} + \nu_0 \qquad (3)$$

where ν_0 is a offset, we could but don't have to determine. This is a relation between the inverse magnetic field and the Landau level number. We extracted the $B_\perp$ fields corresponding to the maxima of the SdH oscillations from the data, as they tell us at what $B_\perp$ fields a Landau level is half filled. The measurement has been made using the sample in second orientation at vanishing parallel field $B_{||} = 0$ and at temperature

TABLE II: This table shows the angle φ in dependency of the desired parallel magnetic field $B_{||}$. The effective angles $\varphi^{\min}$ and $\varphi^{\max}$ mean the angle between which the horizontal rotator has to scan, as the initial position ($B_\perp = 0$) is at $\pi/2$.

$B_{\|\|}$ [T]	φ [°]	$[\varphi^{\min}, \varphi^{\max}]$ [°]
5	30	$[60, 120]$
6	24.62	$[65.38, 114.62]$
7	20.92	$[69.08, 110.92]$
8	18.21	$[71.79, 108.21]$
9	16.13	$[73.87, 106.13]$
10	14.48	$[75.52, 104.48]$
11	13.14	$[76.86, 103.14]$
12	12.02	$[77.98, 102.02]$
13	11.09	$[78.91, 101.09]$
14	10.29	$[79.71, 100.29]$

T=2K. The measurement has been made from $B_\perp = 0$ in both direction $B_\perp < 0$ and $B_\perp > 0$ and the carrier densities we obtained are $p(B_\perp < 0) = 6.063 \cdot 10^{15} \frac{1}{m^2}$ and $p(B_\perp > 0) = 6.102 \cdot 10^{15} \frac{1}{m^2}$. The same can be accomplished using the minima of the SdH oscillations with the difference that at a minimum the Landau level is full/empty while in the maximum case the level is exactly half filled.

B. Method using Hall effect

The second approach uses measurements we've already made but haven't used yet: the Hall effect. The Hall resistance $R_\perp$ and the applied magnetic field $B_\perp$ are connected by a linear relation $R_\perp = r_H B_\perp$, where r_H is the Hall constant and is equal to $r_H = \frac{1}{pe}$. We determine the slope of $R_\perp(B_\perp)$ at $B_\perp = 0$ and find the Hall constant r_H which leads us to the carrier density p_{Hall}. Using the same sample and conditions as before, we obtained for the carrier density using the Hall effect $p_{Hall} = 5.97 \cdot 10^{15} \frac{1}{m^2}$.

C. Carrier density dependence on magnetic field

Curious whether the carrier density changes with variable applied magnetic field B, the carrier densities $p(B)$ for $B = 0, 5, 6, 7, \ldots, 13, 14T$ and at constant temperature $T = 2K$ in second orientation have been determined. The method using the Hall effect had been chosen for this computation, because its faster and simpler than the other method. As one can see in Fig. 6 the carrier density increases until the field B reaches a maximum value at $B \approx 10T$ and decreases again as B tends to higher fields. The carrier density has, in dependency of the applied magnetic field, a value in the range of $[5.95, 6.35] \cdot 10^{15}$.

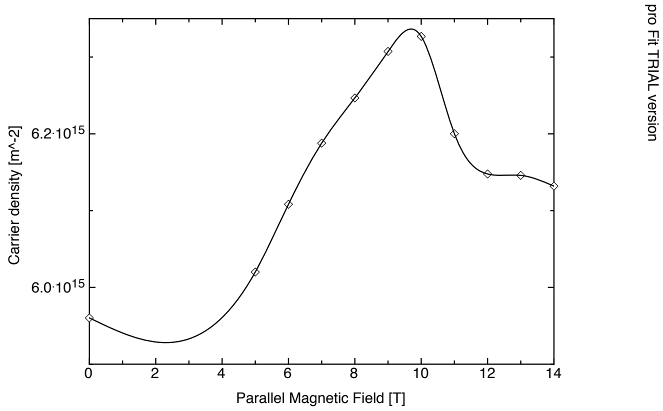

FIG. 6: Dependency of the carrier density with respect to the applied parallel magnetic field $B_{||}$. The curve has been determined using the linear relation between the perpendicular field and the resistance $R_\perp$.

V. OTHER OBSERVATIONS

A. Offset in resistance for different direction

During our measurements we noticed a offset in the resistance at $B_\perp = 0$ of about 300Ω for the two different directions of the sample in the PPMS at an applied field of $10T$. This offset is likely to be produced by the different direction of the parallel field $B_{||}$. In the first case (direction one) the field is in direction of the current where in the second case (second direction) the field is perpendicular to the current (not the sample).

B. Balancing the measurements

Using the horizontal rotator needs a special treatment of the obtained data. As it is not possible to set the absolute angle of the rotator exactly (means that there is always a uncertainty in the absolute angle), we have to correct the difference afterwards during data evaluation. Otherwise the effective mass would depend on the side ($B_\perp \lesseqgtr 0$) we've chosen for determination. One possibility is to plot the resistance with respect to the absolute value of the magnetic field. With this method one can see the difference the n^{th}_{left} maximum on the left ($B_\perp < 0$) and the n^{th}_{right} maximum on the right ($B_\perp > 0$) have and can shift the data half the difference in the respective direction.

Acknowledgments

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Batlogg for giving me the possibility for a slight insight into experimental physics.

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[1] G. I. B. Rössner, D. Chrastina and H. von Känel, *Scattering mechanisms in high-mobility strained Ge channels*, Applied Physics Letters **84**, 3058 (2004).
[2] G. I. B. Rössner and H. von Känel, *Effective mass in remotely doped Ge quantum wells*, Applied Physics Letters **82**, 754 (2003).
[3] `http://www.mathworks.com`.
[4] L. Smrčka and A. Isihara, *Density and magnetic field dependences of the conductivity of two-dimensional electron systems*, J. Phys. C **19**, 6777 (1986).
[5] The goal is to find the envelope of the SdH oscillations. It's obviously not the case that the envelope goes through all maxima and minima of the data. The simplest method to find a good approximation is to look for all the maxima and minima, as they are good estimates for the real tangent points. For other methods and more information see the Appendix A.
[6] `http://www.pnm.ethz.ch`.
[7] Expect from this it is also not the case that the maxima and minima are the tangent points; using the second method lets you "chose" the tangent points of the envelope.
[8] It is important how to go through the maxima and minima. Although it is not necessary that you start with the maxima at the highest magnetic field, it is important that you start defining the maximas, the zero-point and then the minima. The next program used for further analysis depends on the file format.
[9] This interpolation preserves the monotonity of the data points; this is especially helpful for the determination of the effective mass near the edge of the measurement.
[10] This is the only file you have to edit during evaluation. In this file you have to define the paths of the files and the number of maxima and minima for each of the files (see code).

APPENDIX A: DATA EVALUATION USING MATLAB

1. The fitting procedure

After every measurement we start with the data files the PPMS provides. First, one has to extract the data from the file into subfiles for different temperatures and fields (different directories for the applied field and different documents for different temperatures). For each of these data sets we want to find the envelope. This can be done in two different ways. While the first one is simpler, the the second one is safer and more accurate. The first method of getting an envelope is to find all the maxima/minima of the SdH oscillations. This is done by *FindMinMax.m* which lies a cubic spline over the data and searches the obtained spline for points $p(x)$ for which $p(x) > p(y)$ or $p(x) < p(y)$ for any y in the neighborhood of x (where y is a neighbor if $|y - x| = d$, where you can define d) and writes the maxima/minima into a file. The problem using this method is that e.g. for high fields and for the second orientation part of the oscillations don't "end" in a minimum or maximum (and although the oscillations have an envelope). Using *FindMinMax.m* in these cases leads to bad fits for the effective mass[7]. Alternatively one can use *AlternativeEnvelope.m* which uses *ginput* to determine the tangent points of the envelope. You have to define the tangent points by hand starting from the maximum the farest away from $B_\perp = 0$ going in direction of $B_\perp = 0$ then returning again defining the minima[8]. The next task is to find the amplitude of the SdH oscillations at a specific magnetic field B. The program *effectMass.m* takes a field B, the path to the file where the maxima and minima are stored, the number of maxima and minima in the file and calculates the amplitude at magnetic field B. This is done by using a Piecewise Cubic Hermite Interpolating Polynomial (PCHIP[9]) for both the maxima and minima. As one amplitude at one temperature is not helpful at all, the program *FitEffMass.m* does everything for you. You can pass a vector x of different magnetic fields as input and the program returns the effective mass (as a vector) for every field. *FitEffMass.m* calls *SdHAmplitudes.m* where all information is stored[10] and gets all the amplitudes for the different temperatures back. The fit is "non linear least square". For slowly converging data the 'MaxFunEvals' and 'MaxIter' have to be increased. The *FitEffMass.m* returns the vector ψ of effective masses; plotting (x, ψ) displays the effective mass with respect to the perpendicular magnetic field.

2. FindMinMax.m - Finds minima and maxima

```
function FindMinMax(x_Axis,y_Axis,dest,exact,from,to)

%--------------------------------------------------%
% This application finds the maxima/minima of
% a given data set and writes them into a file
% specified by 'dest'. Inputs are the two data
% sets (x_Axis, y_Axis), the destination 'dest',
```

```
% the step width 'exact' for the maxima/minima iteration
% and the range [from,to].
%
% Thomas Bisig (07.2005), thomas@macapp.net
%--------------------------------------------------%

% Redefining the constants
x = x_Axis;
y = y_Axis;
d = dest;
diff = exact;

% Maxima/Minima array
mini = [];
maxi = [];

% Array of the corresponding values F(mini/maxi)
mini_v = [];
maxi_v = [];

% Fit cubic spline to data
F = fit(x,y,'cubicspline');

% Parse the whole spline in search of
% local maxima/minima
for r = from:diff:to
    if (r~=0)
    % Loop for the maxima
       if (F(r-diff)<F(r) && F(r+diff)<F(r))
         maxi = [maxi r];
        maxi_v = [maxi_v F(r)];
      end
    % Loop for the minima
       if (F(r-diff)>F(r) && F(r+diff)>F(r))
         mini = [mini r];
        mini_v = [mini_v F(r)];
     end
    end
end

% Definitions for later purpose
Header = '[HEADER]\n';
Description = 'Here comes your description of the file''s content\n\n';
Body = '[DATA]\n';
Axis = 'Write here what each column stands for\n';

% Sorting the data in a manner we can reuse
vect = [maxi 0 mini];
vect_v = [maxi_v F(0) mini_v];
finaldata = [vect; vect_v];

% Write everything to file 'dest'
fid = fopen(dest,'w');
fprintf(fid,Header);
fprintf(fid,Description);
fprintf(fid,'Number of maxima: %d\nNumber of minima: %d\n\n',length(maxi),length(mini));
fprintf(fid,Body);
fprintf(fid,Axis);
```

```
fprintf(fid,'%e %e\n',finaldata);
fclose(fid);
```

3. AlternativeEnvelope.m - Alternative way of finding minima and maxima

```
function AlternativeEnvelope(x2,x3,x4,x5,x6,p2,p3,p4,p5,p6)

%--------------------------------------------------%
% As the FindMaxMin.m does not work all the time (due
% to fastly decreasing amplitudes) this application
% uses ginput. You have to click on the points you want
% the envelope go through and the application writes these
% points to a file MinMax_i. (Additionally one can edit the
% number of maxima and minima in these files. (Later you need
% them anyway). Inputs are the position (magnetic field) p_i and
% the resistances x_i.
%
% THIS IS JUST A SHORT HACK. Make the code look better by using
% a FOR loop or anything you want.
%
% Thomas Bisig (07.2005), thomas@macapp.net
%--------------------------------------------------%

plot(p2,x2);
[a2,b2]=ginput;
plot(p3,x3);
[a3,b3]=ginput;
plot(p4,x4);
[a4,b4]=ginput;
plot(p5,x5);
[a5,b5]=ginput;
plot(p6,x6);
[a6,b6]=ginput;

e2 = [a2'; b2'];
e3 = [a3'; b3'];
e4 = [a4'; b4'];
e5 = [a5'; b5'];
e6 = [a6'; b6'];

% Definitions for later purpose
Header = '[HEADER]\n';
Description = 'Here comes your description of the file''s content\n\n';
Body = '[DATA]\n';
Axis = 'Write here what each column stands for\n';

fid = fopen('MinMax2.dat','w');
fprintf(fid,Header);
fprintf(fid,Description);
fprintf(fid,'Number of maxima: %d\nNumber of minima: %d\n\n','?','?');
fprintf(fid,Body);
fprintf(fid,Axis);
fprintf(fid,'%e %e\n',e2);
fclose(fid);

fid = fopen('MinMax3.dat','w');
fprintf(fid,Header);
```

```
fprintf(fid,Description);
fprintf(fid,'Number of maxima: %d\nNumber of minima: %d\n\n','?','?');
fprintf(fid,Body);
fprintf(fid,Axis);
fprintf(fid,'%e %e\n',e3);
fclose(fid);

fid = fopen('MinMax4.dat','w');
fprintf(fid,Header);
fprintf(fid,Description);
fprintf(fid,'Number of maxima: %d\nNumber of minima: %d\n\n','?','?');
fprintf(fid,Body);
fprintf(fid,Axis);
fprintf(fid,'%e %e\n',e4);
fclose(fid);

fid = fopen('MinMax5.dat','w');
fprintf(fid,Header);
fprintf(fid,Description);
fprintf(fid,'Number of maxima: %d\nNumber of minima: %d\n\n','?','?');
fprintf(fid,Body);
fprintf(fid,Axis);
fprintf(fid,'%e %e\n',e5);
fclose(fid);

fid = fopen('MinMax6.dat','w');
fprintf(fid,Header);
fprintf(fid,Description);
fprintf(fid,'Number of maxima: %d\nNumber of minima: %d\n\n','?','?');
fprintf(fid,Body);
fprintf(fid,Axis);
fprintf(fid,'%e %e\n',e6);
fclose(fid);
```

4. effectMass.m - Amplitudes of the SdH oscillations

```
function difference=effectMass(B,path,max,min)

%--------------------------------------------------------%
% Input the desired magnetic Field B and the
% program calculates the amplitudes of the
% Shubnikov-de Haas Oscillations. 'path' is the
% file to read, max (min) is the number of maxima (minima)
% in the data file.
%
% Thomas Bisig (07.2005), thomas@macapp.net
%--------------------------------------------------------%

% MATLAB starts counting with 1 not 0
max = max + 1;
nrlines = max + min;

% Open a file with data and reading from line nine $nrlines lines
% Delimiter is a ',' and the numbers are read as double.
fi = fopen(path,'r');
T = textscan(fi, '%n %n', nrlines, 'headerLines', 8, 'delimiter', ',');
fclose(fi);
```

```
% Matlab uses cell classes to store the text scans.
% We transform this into a matrix Z (more comfortable)
% and split the magnetic field (M) from the resistivity (R).
Z = cell2mat(T);

M = Z(:,1);
R = Z(:,2);

% We have to differ between maxima (1) and minima (2)
% of magnetic field and resistivity.
M1 = M(1:max);
M2 = M(max:nrlines);
R1 = R(1:max);
R2 = R(max:nrlines);

% To find the envelope we have to calculate
% a cubic spline through the maxima (minima) points.
% This is not the best choice for a fit. Use the
% pchip iteration. read 'help pchip' for more
% information.
% F1 = fit(M1,R1,'cubicspline'); % Maxima
% F2 = fit(M2,R2,'cubicspline'); % Minima

F1 = interp1(M1,R1,'pchip','pp'); % Maxima
F2 = interp1(M2,R2,'pchip','pp'); % Minima


%We are interested in the difference (amplitude) of
% the Shubnikov-de Haas oscillations.
% difference = F1(B)-F2(B);

difference = ppval(F1,B)-ppval(F2,B);
```

5. FitEffMass.m - Main program; finds effective mass

```
function RatioToElectron = FitEffMass(B)

%-----------------------------------------------------%
% Input is the magnetic field B and the program
% computes a non linear least square fit for the
% amplitudes of the Shubnikov-de Haas oscillations
% calculated by SdHAmplitudes().
%
% REQUIRED FILES: SdHAmplitudes.m, effectMass.m
%
% Thomas Bisig (07.2005), thomas@macapp.net
%-----------------------------------------------------%

%Defining the constants we need
e = 1.602e-19;
k = 1.380e-23;
h = 1.055e-34;
me = 9.11e-31;
l = length(B);

for j=1:1:l
```

```
    konst = 2*pi*pi*k./(h*e*B(j)) * me;

    % This is our x-axis (T=2,...,6K)
    a=[2 3 4 5 6]';

    % Options for the fit: We start at C=1000 and n=0.1
    % and the two values shall not go lower than 0.
    opts = fitoptions('Method','NonlinearLeastSquares','MaxFunEvals',10000,'MaxIter',2000);
    opts.startpoint = [1000 0.1];
    opts.lower = [0 0];

    % Fitting our defined equation to the data set
    m = fittype('C*konst*n*x./sinh(konst*n*x)','problem','konst');
    f = fit(a, SdHAmplitudes(B(j))', m, 'problem', konst, opts);

    % Printing the effective mass
    % (Uncomment this to use it. Change
    % the function definition as well)
    %m_eff = me * f.n

    % Prints the ratio between the
    % electron mass and the computed
    % effective mass.
    RatioToElectron(j) = f.n;
end
```

6. SdHAmplitudes.m - Sub-application; edit data here

```
function d = SdHAmplitudes(B)

%-----------------------------------------------------------------%
% This little application is used as a sub-application. It's rather
% a 'specify your data' file as it basically calls effectMass.m for
% the different files.
%
% REQUIRED FILES: effectMass.m
%
% YOU HAVE TO EDIT THIS FILE TO YOUR NEEDS
% Define the paths of the different data files in 'Datafile(n,1)'
% and the number of maxima (minima) in max[] (min[]).
%
% Thomas Bisig (07.2005), thomas@macapp.net
%-----------------------------------------------------------------%


% ---------------------------- SPECIFIC DATA (START) ----------------------------
% Number of files to look at
nrtemps = 5;

% Which data do you want to consider?
Datafile(1,1).name = 'D:\Semesterarbeit Thomas\1. Orientierung\Bp=var\140000\MinMax2.dat';
Datafile(2,1).name = 'D:\Semesterarbeit Thomas\1. Orientierung\Bp=var\140000\MinMax3.dat';
Datafile(3,1).name = 'D:\Semesterarbeit Thomas\1. Orientierung\Bp=var\140000\MinMax4.dat';
Datafile(4,1).name = 'D:\Semesterarbeit Thomas\1. Orientierung\Bp=var\140000\MinMax5.dat';
Datafile(5,1).name = 'D:\Semesterarbeit Thomas\1. Orientierung\Bp=var\140000\MinMax6.dat';

% Amount of maxima/minima for each file/temperature
```

```
max = [7 7 7 7 6];
min = [8 7 7 6 6];

% ---------------------------- SPECIFIC DATA (END) ---------------------------------


% Looping through all the nrtemps different temperatures
for i = 1:nrtemps
    path = getfield(Datafile, {i,1}, 'name');
    d(i) = effectMass(B,path,max(i),min(i));
end
```